Persuasion under Insufficient Reason

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Introduction

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- How to choose among these models?

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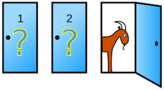
Bayesian approach



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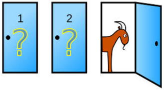

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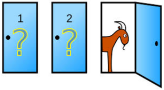

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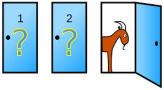

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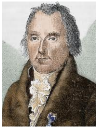

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3. Agent observes t and m, and then makes his choice between $a = 0$ and $a = 1$.

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Definition

A preference relation $\succeq$ admits a <u>vNM expected utility representation</u> if there exists a utility function $U : X \to \mathbb{R}$ such that for each $p_1, p_2 \in \Delta(X)$

$$p_1 \succeq p_2 \quad \text{if and only if} \quad \int_{x \in X} U(x) \mathrm{d}p_1(x) \geq \int_{x \in X} U(x) \mathrm{d}p_2(x).$$

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A preference relation $\succeq$ has a vNM expected utility representation if and only if it has a lottery comparison representation.

Moreover, if a vNM utility U and a benchmark lottery b both represent $\succeq$, then there exist $\alpha \in \mathbb{R}$ and $\beta > 0$ such that

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- We can now treat $U(x)$ as a probability distribution

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  - $H(g, u) = H(g) + H(u)$

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The maximum-entropy utility in the class of CARA, CRRA, or HARA is risk neutral.

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- In what follows, assume risk neutral utility

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    - Persuasion problem is linear because maximum entropy U is linear
    - In a linear problem with uniform distribution, every signal is optimal.

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    - If the density is log-concave (log-convex) then upper (lower) censorship is optimal

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  - What about other summary statistics?
  - What if we observe the mean with, e.g., left censoring?
  - What are naturally occurring summary statistics about risk attitude?