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Optimal mechanisms for an auction mediator[☆]

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ABSTRACT

We consider a dynamic auction environment with a long-lived seller and short-lived buyers mediated by a third party. A mediator has incomplete information about traders' values and selects an auction mechanism to maximize her expected revenue. We characterize mediator–optimal mechanisms and show that an optimal mechanism has a simple implementation as a Vickrey auction with a reserve price where the seller pays to the mediator only a fixed percentage from the closing price.

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1. Introduction

This paper considers a model of Internet-style trade, where a seller cannot deal directly with buyers, and the trade must be mediated. A mediator sets the rules of a trade procedure (an auction mechanism) and can collect fees from the traders. Our main question is:

What auction mechanisms maximize the mediator's revenue?

This question has not only theoretic interest, but also practical relevance, as we attempt to capture, in a stylized way, an interaction between sellers and buyers on Internet auctions where the role of mediators is played by such giant commercial institutions as *eBay*, the dominant auction site in many countries with reported revenue steadily growing (despite the recent financial crisis) and reaching $8.7 billion in 2009,[1] and its former major competitors, *Amazon* and *Yahoo*.[2]

Consider a setting with a seller who has a single object for sale, a large population of buyers, and a mediator. The seller and buyers have independent private values for the object. In the initial period the mediator announces auction rules. That is, she chooses an auction mechanism through which she collects a part of the trade surplus.[3] The seller observes the mechanism and decides whether to consume the object or to put it for sale. If the object is consumed, the game ends. If the object is put for sale in period $t \geq 1$, a set of n buyers is drawn at random from the buyers' population and the auction takes place (in every period a new sample of buyers is drawn).

We characterize optimal mechanisms for a mediator. Furthermore, we demonstrate that an optimal mechanism admits a simple and practical implementation as a repeated Vickrey auction where the seller pays to the mediator a *closing fee*, that is, a fixed percentage of the final price. This is in contrast to Myerson and Satterthwaite (1983) who analyze a *single-period* bilateral trade mediated by a "broker". Myerson and Satterthwaite's (1983) optimal mechanism is a nontrivial function of the seller's report about his private type and hence it lacks a simple implementation.

We search for an optimal mechanism on the class of stationary ones (fixed over time). This assumption is motivated by practical concerns of equal treatment or non-discrimination, that is to say, the same auction rules must apply for all participants, irrespective of their identity or period of participation. The real life supports this

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[1] See eBay press releases on http://investor.ebay.com.

[2] *Yahoo* discontinued its Internet auction service on June 16, 2007 and *Amazon* on September 8, 2008.

[3] By an auction mechanism we understand a one-period game with incomplete information set up by the mediator and played by the traders in which a desirable outcome obtains in a Bayesian Nash equilibrium.

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assumption: in almost all Internet auctions the rules are fixed. An additional reason for focusing on stationary mechanisms is that optimality in such mechanisms can be obtained endogenously under a simple assumption of *anonymity* of sellers. This assumption requires that a seller can freely leave and re-enter an auction site under a different identity (as a newcomer). We argue that it allows finding an optimal mechanism among stationary ones. Notice that anonymity immediately entails that the *seller's expected continuation payoff from future auctions must be non-decreasing over time*. Otherwise the seller could be better off by withdrawing the object and starting a new auction instead (pretending to be a different seller, which is possible by the anonymity assumption). On the other hand, a continuation payoff for a seller which is increasing over time cannot be optimal either. The seller's continuation payoff is important only for the seller's participation decision. So, since a seller has agreed to participate for some continuation payoff, he would definitely participate for the same payoff in any future period, hence it cannot be optimal to pay him more. It follows that in an optimal mechanism the seller's continuation payoff is constant over time, and, consequently, it is sufficient to concentrate on stationary mechanisms only.

There are two other important assumptions in our model. First, whenever the seller fails to sell the object, he is allowed to offer it for (re-)auction again, as many times as he wants. This assumption is realistic for Internet auctions. Indeed, a seller has a re-auction option in real life and this option has essential impact on players' strategic behavior, as noted, for example, by Fudenberg et al. (1985), Milgrom (1987), McAfee and Vincent (1997), Horstmann and LaCasse (1997), Gupta and Lebrun (1999). Our second assumption is that in every trade the seller faces a different set of buyers drawn from a large population. This is reasonable in the context of Internet auctions where a typical auction runs several days and most of the bids are received in the very last day. Our model can be considered as an instance in many similar sales on Internet where a buyer's objective is to purchase an object of a certain kind, not to purchase an object from a specific seller. A buyer who fails to buy an object from a seller can obtain it elsewhere and therefore has no reason to return to this particular seller. In contrast, the existing literature on auctions with resale assumes that there is the same set of bidders in all auctions.[4] This implies two differences from our model. In models with a possibility of one-time after-auction resale, each bidder places a positive probability on buying in a secondary market if she loses the auction (Gupta and Lebrun, 1999; Haile 1999, 2000, 2001, 2003; Zheng 2002; Krishna, 2002, Section 4.4; Calzolari and Pavan 2006; Garratt and Tröger, 2006; Pagnozzi 2007). In models with re-auctioning, the optimal reserve price declines due to Bayesian updating of the distribution of bidders' private values after every auction (Fudenberg et al., 1985; McAfee and Vincent 1997).

In our model a winning bidder is not allowed to re-auction the object. This is a simplifying assumption which can be relaxed without any effect on the results since a new set of bidders arrives in each period, there is no issue of signaling and information communication for the bidders between periods (in contrast to Haile 1999, 2000, 2001, 2003; Zheng 2002; and others). If a winning bidder becomes a seller, she would face ex-ante the same environment in the next-period auction. The expected revenue from a new auction is not higher than her current use value, thus she prefers to consume the object. This contrasts our results, in particular, to Zheng (2002) who assumes that a fixed, finite set of bidders is involved in trade, where, despite that bidders are ex-ante symmetric, the initial seller and the winning bidder face different trade environments, and the winner may benefit from a re-auction.

[4] The exceptions are Haile (1999, 2001) who allows new bidders (in particular, *all* new bidders) to participate in a re-auction; and Bikhchandani and Huang (1989), Bose and Deltas (1999, 2007) and Calzolari and Pavan (2006) who model resale to a given secondary market where the original bidders need not participate.

Our paper is related to Jullien and Mariotti (2006) who study an interaction between a seller and a few buyers in a similar setting, but with a common value component in traders' utilities for the object. Jullien and Mariotti (2006) focus on efficiency issues and find that trade mediated by an uninformed "broker" may be more efficient than unmediated trade.

Matros and Zapechelnyuk (2008) consider a problem similar to the current paper, but they focus on a very restricted set of auction mechanisms, Vickrey auctions where the mediator chooses two fees, a *listing fee*, a fixed amount paid by a seller regardless of the auction outcome, and a *closing fee*, a percentage of the closing price if the object is sold. This paper generalizes Matros and Zapechelnyuk (2008) to a general class of auction mechanisms.

The paper is organized as follows. The model is described in Section 2. We analyze the seller's optimal participation decision in Section 3 and characterize mediator–optimal mechanisms in Section 4. Section 5 describes a simple implementation of an optimal mechanism. Section 6 concludes. The Appendix contains omitted proofs.

2. The model

Let player 0 be a seller and let $\mathcal{N}$ be a large (infinite) population of bidders. The seller has one object for sale. Let v_0 be a private *use value* of the seller and v_i be a private *use value* of bidder $i \in \mathcal{N}$. Assume that all use values are independent, furthermore, bidders' use values are identically distributed on interval $[\underline{v}, \bar{v}]$ according to distribution function F, and the seller's use value is distributed on the same interval according to distribution function H. We also assume that functions F and H are differentiable and have positive density on $(\underline{v}, \bar{v})$, and, in addition, satisfy the monotonic hazard rate conditions (e.g., Myerson 1981), that is, $z - \frac{1-F(z)}{f(z)}$ and $z + \frac{H(z)}{h(z)}$ are strictly increasing on $(\underline{v}, \bar{v})$, where f and h denote the corresponding density functions. Distribution functions F and H are common knowledge, and all players are risk neutral.

The timing of the game is as follows. In period $t = 0$, the mediator announces an auction mechanism that will be used in all further interactions. In period $t = 1, 2, \ldots$, the seller either consumes the object (and the game ends) or puts it for sale via the specified auction mechanism. Then a random sample of n bidders[5] is selected from population $\mathcal{N}$, the object is allocated and the payments are transferred according to the mechanism. If the object is allocated to one of the bidders, the game ends. Otherwise, if the object is returned to the seller, the game proceeds to the next period.

Without loss of generality, we consider the class of direct mechanisms (e.g., Myerson, 1981). In a direct mechanism the seller and each bidder simultaneously and confidentially report their use values to the mediator, and the mediator then determines who gets the object and how much each trader must pay (or receive) as some functions of the vector of reported use values. Formally, a direct mechanism is a pair $(\mathbf{p}, \mathbf{x})$ where[6] $\mathbf{p} : [\underline{v}, \bar{v}]^{n+1} \to \Delta^{n+1}$ describes probabilities of various outcomes and $\mathbf{x} : [\underline{v}, \bar{v}]^{n+1} \to \mathbb{R}^{n+1}$ describes payments of the traders as functions of their *reported* use values. Namely, given the vector of reports at period t, $\mathbf{w}^t = (w_0^t, w_1^t, \ldots, w_n^t)$, $p_i(\mathbf{w}^t)$ is the probability that bidder i gets the object, $i = 1, \ldots, n$, $p_0(\mathbf{w}^t) = 1 - \sum_{i=1}^n p_i(\mathbf{w}^t)$ is the probability that the seller retains the object; $x_i(\mathbf{w}^t)$ is a payment of bidder $i = 1, \ldots, n$ to the mediator, and $x_0(\mathbf{w}^t)$ is a payment of the mediator to the seller. Note that for every

[5] The results can be generalized to the case where the number of bidders, n, is random, drawn from the same distribution in each period. Indeed, all what matters here is that a seller makes the decision of auctioning his object *before* n is drawn, thus his decision depends on the distribution of the number of bidders (which is constant across periods), but not on its realizations.

[6] Δ^{n+1} denotes the unit simplex in $(n+1)$-dimensional space.

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$i = 1,\ldots,n$, x_i is allowed to be non-zero even if bidder i does not receive the object.

In every period, due to our assumption that a new set of n random bidders is drawn, the seller faces ex-ante the same problem: auctioning an object via a fixed mechanism to a set of n bidders with private use values independently drawn from interval $[\underline{v},\bar{v}]$ with distribution function F. It follows that a decision of the seller that is optimal at period t should be also optimal at every other period, before or after t. Thus we focus only on players' Markov strategies that depend on traders' private use values and do not depend on the information available from previous transactions. In addition, we assume that bidders are anonymous, that is, a mechanism cannot depend on bidders' identities, and we focus on symmetric strategies for the bidders.

Formally, a *symmetric Markov strategy of a bidder*, $\omega : [\underline{v},\bar{v}] \to [\underline{v},\bar{v}]$, is her reported value as a function of her actual use value. A *Markov strategy of the seller* is a pair (α, ω_0), where $\alpha : [\underline{v},\bar{v}] \to [0,1]$ specifies the probability, $\alpha(v_0)$, that the seller decides to auction the object, and $\omega_0 : [\underline{v},\bar{v}] \to [\underline{v},\bar{v}]$ specifies his reported value, $\omega_0(v_0)$, as functions of his use value v_0. We refer to component α as the *participation strategy* of the seller. A Bayesian Nash equilibrium of this game when every player uses a Markov strategy is called a *Markov perfect equilibrium* and described by a triple $(\alpha, \omega_0, \omega)$.

The following lemma is a standard result (e.g., Krishna, 2002) that shows that without loss of generality we can restrict attention to *direct truthful mechanisms*, that is, the mechanisms where reporting true use values is a Markov perfect equilibrium.

Lemma 1. (Revelation Principle) *Given a mechanism* $(\mathbf{p},\mathbf{x})$ *and a Markov perfect equilibrium* $(\alpha, \omega_0, \omega)$ *of the correspondent game, there exists a direct truthful mechanism* $(\mathbf{p}', \mathbf{x}')$ *which has a payoff-equivalent Markov perfect equilibrium* $(\alpha, \omega_0', \omega')$ *such that* $\omega_0'(v_0) = v_0$ *and* $\omega'(v_i) = v_i$, $i = 1,\ldots,n$.

Proof. For every $(v_0, v_1, \ldots, v_n) \in [\underline{v},\bar{v}]^{n+1}$ define $\mathbf{p}'(v_0, v_1, \ldots, v_n) = \mathbf{p}(\omega_0(v_0), \omega(v_1), \ldots, \omega(v_n))$
and $\mathbf{x}'(v_0, v_1, \ldots, v_n) = \mathbf{x}(\omega_0(v_0), \omega(v_1), \ldots, \omega(v_n))$.□

Note that for non-Markov perfect equilibria the revelation principle need not hold, since an equilibrium strategy for the seller may stipulate reporting different values in different periods, which in general cannot be mapped into a strategy of reporting truth in all periods.

Let us introduce some more notations. Let $\mathbf{V} = [\underline{v},\bar{v}]^{n+1}$ be the set of type profiles of the seller and bidders $1,\ldots,n$, and let $\mathbf{V}_{-i} = [\underline{v},\bar{v}]^n$ be the set of type profiles of all players except i, $i = 0,1,\ldots,n$. Denote by $\mathbf{v}$ and $\mathbf{v}_{-i}$ generic elements of $\mathbf{V}$ and $\mathbf{V}_{-i}$, and denote by $\mathbf{f}$ and $\mathbf{f}_{-i}$ the joint densities of types in $\mathbf{V}$ and $\mathbf{V}_{-i}$, respectively. Next, for every $i = 0,1,\ldots,n$ denote by $\overline{p}_i(v_i)$ the probability of i to obtain (retain for $i = 0$) the object, conditional on i's use value v_i,

$$\overline{p}_i(v_i) = \int_{\mathbf{V}_{-i}} p_i(v_i, \mathbf{v}_{-i}) \mathbf{f}_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}.$$

Also, denote by $x_i(v_i)$ the expected payment of bidder i to the mediator (from the mediator to the seller for $i = 0$) conditional on v_i,

$$\overline{x}_i(v_i) = \int_{\mathbf{V}_{-i}} x_i(v_i, \mathbf{v}_{-i}) \mathbf{f}_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}.$$

Then, the expected utility of bidder $i = 1,\ldots,n$ is defined by the following:

$$U_i(v_i) = v_i \overline{p}_i(v_i) - \overline{x}_i(v_i). \tag{1}$$

The expected seller's utility is defined by the following:

$$U_0(v_0) = (1 - \alpha(v_0))v_0 + \alpha(v_0)(\overline{x}_0(v_0) + \overline{p}_0(v_0)\delta U_0(v_0)), \tag{2}$$

where δ is a discount factor, $0 < \delta < 1$. Thus, with probability $1 - \alpha(v_0)$ the seller consumes the object and obtains its use value v_0, and with probability $\alpha(v_0)$ he auctions the object and obtains the expected transfer $x_0(v_0)$ from the mediator and, if the object is not sold, the discounted next-period expected utility.

A direct truthful mechanism $(\mathbf{p},\mathbf{x})$ is *feasible* if it satisfies the following constraints:

(a) *Individual rationality*. For every trader $i = 0,1,\ldots,n$ and every $v_i \in [\underline{v},\bar{v}]$

$$U_i(v_i) \geq 0. \tag{3}$$

(b) *Incentive compatibility: Reporting true use values is a Nash equilibrium*. For every trader $i = 0,1,\ldots,n$ and all $v_i, w_i \in [\underline{v},\bar{v}]$

$$U_i(v_i) \geq U_i(w_i \mid v_i), \tag{4}$$

where $U_i(w_i|v_i)$ is the expected utility of trader $i = 0,1,\ldots,n$ if she reports w_i when her true use value is v_i. More specifically, for each $i = 1,\ldots,n$

$$U_i(w_i \mid v_i) = v_i \overline{p}_i(w_i) - \overline{x}_i(w_i) \tag{5}$$

and

$$U_0(w_0 \mid v_0) = (1 - \alpha(v_0))v_0 + \alpha(v_0)(\overline{x}_0(w_0) + \overline{p}_0(w_0)\delta U_0(v_0)).$$

Note that the seller's next-period expected revenue $U_0(v_0)$ does not depend on the current report w_0.

3. Seller's participation strategy

Let us now describe the optimal participation strategy for the seller, α. The seller decides to participate, i.e., to auction the object, if and only if his expected payoff from auctioning the object exceeds his use value v_0. Note that, since a new set of bidders arrives in each period, the seller faces ex-ante the same problem in every period. Stationarity of the environment implies that if the seller decides to auction the object in the first period, he should re-auction it forever, until it is sold.

Denote by $U_0^{(\mathbf{p},\mathbf{x})}$ the maximum expected revenue of the seller if he auctions the object in all periods (i.e., for $\alpha(v_0) = 1$),

$$U_0^{(\mathbf{p},\mathbf{x})} = \max_{w_0 \in [\underline{v},\bar{v}]} \left[\overline{x}_0(w_0) + \overline{p}_0(w_0)\delta U_0^{(\mathbf{p},\mathbf{x})}\right]. \tag{6}$$

Observe that $U_0^{(\mathbf{p},\mathbf{x})}$ does not depend on v_0, since the object is never consumed.[7] In equilibrium, the seller will auction the object whenever his maximum gain from participation is greater than his use value. Therefore, the equilibrium strategy α must satisfy for every $v_0 \in [\underline{v},\bar{v}]$

$$\alpha(v_0) = \begin{cases} 1, & \text{if} \quad U_0^{(\mathbf{p},\mathbf{x})} \geq v_0, \\ 0, & \text{if} \quad U_0^{(\mathbf{p},\mathbf{x})} < v_0. \end{cases}$$

The case $U_0^{(\mathbf{p},\mathbf{x})} = v_0$ is a zero probability event, thus without any effect on the results we can assume $\alpha(v_0) = 1$ for that case.

Note that by the incentive compatibility constraint (4), the seller's payoff is maximized when he reports his true use value, $w_0 = v_0$. Thus we can rewrite (6) as follows,

$$U_0^{(\mathbf{p},\mathbf{x})} = \overline{x}_0(v_0) + \overline{p}_0(v_0)\delta U_0^{(\mathbf{p},\mathbf{x})}. \tag{7}$$

[7] $U_0^{(\mathbf{p},\mathbf{x})}$ may depend on v_0 if there is a small probability that the seller is not able to re-auction the object and thus required to consume it. See Section 5 for a discussion.

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As observed, $U_0^{(\mathbf{p},\mathbf{x})}$ does not depend on v_0, and hence the right-hand side of (7) does not depend on v_0 either (though v_0 in $\overline{x}_0(v_0)$ and $\overline{p}_0(v_0)$ will be retained for consistency of notations).

4. Mediator–optimal mechanisms

We now find an auction mechanism that is optimal for the mediator on the set of all feasible direct truthful mechanisms, denoted by $\mathcal{M}$.

Given a direct truthful mechanism $(\mathbf{p},\mathbf{x})$ and a seller's participation strategy α, the expected utility of the mediator is defined as follows,

$$U_M = \int_{\mathbf{V}} \alpha(v_0)\left(\sum_{i=1}^{n} x_i(\mathbf{v}) - x_0(\mathbf{v}) + p_0(\mathbf{v})\delta U_M\right)\mathbf{f}(\mathbf{v})d\mathbf{v}. \tag{8}$$

For a given realization of traders' use values, $\mathbf{v}$, the mediator's revenue from the auction is given by the expression in parentheses and equal to the sum of payments from the bidders net of the payment to the seller, plus the expected next-period gain if the object is not sold. Note that this revenue is collected on only under the condition that the seller is willing to participate, $\alpha(v_0)=1$, i.e., his own expected revenue is greater than v_0. Consequently, the key to the optimization problem for the mediator lies in balancing two opposite forces: the net revenue of the mediator conditional on the seller's participation and the likelihood that the seller decides to participate.

Before we turn to balancing these two forces, it will be convenient first to solve an auxiliary problem. Fix the seller's expected revenue U_0 from auctioning the object and find a mediator–optimal mechanism among those that yield expected revenue U_0 to any seller. This is equivalent to a mechanism where the mediator acquires the object from the seller for price U_0 and then auctions it off.

Formally, for every U_0^* let $\mathcal{M}(U_0^*)$ be the set of mechanisms where every seller's expected revenue is exactly U_0^*,

$$\mathcal{M}\left(U_0^*\right) = \left\{(\mathbf{p},\mathbf{x})\in\mathcal{M} : U_0^{(\mathbf{p},\mathbf{x})} = U_0^*\right\}.$$

We now find the mediator–optimal mechanism on $\mathcal{M}(U_0^*)$ whenever this set is nonempty.

Let $(\mathbf{p},\mathbf{x})\in\mathcal{M}\left(U_0^*\right)$. Fix the seller's use value v_0, and suppose that $v_0\le U_0^*$, that is, the seller always auctions the object. Conditional on this event, the expected revenue of the mediator is given by

$$U_M(v_0) = \int_{\mathbf{V}_{-0}} \sum_{i=1}^{n} x_i(\mathbf{v})\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0} - \overline{x}_0(v_0) + \overline{p}_0(v_0)\delta U_M(v_0), \tag{9}$$

and the total revenue of the seller and the mediator is given by

$$\begin{aligned} Z(v_0) &= \int_{\mathbf{V}_{-0}} \sum_{i=1}^{n} x_i(\mathbf{v})\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0} + \overline{p}_0(v_0)\delta Z(v_0) \\ &\equiv U_M(v_0) + U_0^*. \end{aligned} \tag{10}$$

Denote by $C(v_i)$ the *virtual value* of bidder i, $i=1,\ldots,n$,

$$C(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}. \tag{11}$$

The difference $v_i - C(v_i)$ is referred in the literature as the *information rent* of bidder i (see the discussion in Krishna 2002, Section 5.2.3).

The next lemma states that an optimal mechanism is achieved by allocating the object to the bidder with the highest virtual value whenever it is greater than the total expected continuation revenue of the mediator and the seller, and otherwise returning the object to the seller.

Lemma 2. *A mechanism $(\mathbf{p},\mathbf{x})$ is mediator–optimal on $\mathcal{M}(U_0^*)$ if the following holds.*

(i) *The total revenue of the seller and the mediator, $Z(v_0)$, conditional on $v_0\le U_0^*$ is independent of v_0 and equal to Z^*, where Z^* is a unique solution of the equation*

$$Z^* = \int_{\mathbf{V}_{-0}} \max\left\{\delta Z^*, \max_{i=1,\ldots,n} C(v_i)\right\}\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0}. \tag{12}$$

on $\left[\underline{v}, \overline{v}\right]$;

(ii) *the allocation rule $\mathbf{p}$ satisfies*

$$\mathbf{p}(\mathbf{v})\in \underset{p'\in\Delta^{n+1}}{\operatorname{argmax}}\left\{p_0'\delta Z^* + \sum_{i=1}^{n} p_i'C(v_i)\right\}, \quad \mathbf{v}\in\mathbf{V};$$

(iii) *the transfer rule $\mathbf{x}$ satisfies*

$$\overline{x}_0(v_0) = U_0^*(1-\delta\overline{p}_0(v_0)), \quad \text{and}$$
$$\overline{x}_i(v_i) = v_i\overline{p}_i(v_i) - \int_{\underline{v}}^{v_i} \overline{p}_i(z)f(z)dz, \quad i=1,\ldots,n.$$

The proof is deferred to the Appendix. Conditions (ii) and (iii) say that a mechanism $(\mathbf{p},\mathbf{x})$ that satisfies the conditions of Lemma 2 is the Myerson's (1981) optimal mechanism, with the seller's "outside option" equal to δZ^* and with an additional element, the transfer between the seller and the mediator, x_0. Condition (i) says that the joint revenue of the seller and the mediator, Z^*, is determined as the unique solution of a dynamic maximization problem. Note that, though we find optimal mechanisms on a set $\mathcal{M}(U_0^*)$ that depends on the expected revenue U_0 of the seller, the only part of the mechanism that is a function of U_0^* is x_0. The total expected revenue Z^* of the mediator and the seller, as well as the allocation rule $\mathbf{p}$ and the transfers from the bidders $\mathbf{x}_{-0}$ are independent of U_0^*.

It remains to determine the optimal value of U_0^* and then to choose x_0 that satisfies (iii) in Lemma 2. Observe that for every U_0^* the expected revenue of the mediator in an optimal mechanism on $\mathcal{M}(U_0^*)$ is given by the following:

$$U_M = \left(Z^* - U_0^*\right)H\left(U_0^*\right).$$

Here, $Z^* - U_0^*$ is the mediator's revenue conditional on the seller's participation (otherwise it is equal to zero) and $H(U_0^*)$ is the probability that the seller participates, $H(U_0^*) = Pr[v_0\le U_0^*]$.

Consider a mechanism $(\mathbf{p},\mathbf{x})$ that achieves the total expected revenue of Z^* for the seller and the mediator. While keeping $\mathbf{p}$ and $\mathbf{x}_{-0}$ the same, the seller's expected revenue U_0 can take any value in $[0,Z^*]$. To see this, let $x_0(\mathbf{v}) = \lambda\sum_{i=1}^{n} x_i(\mathbf{v})$, $\mathbf{v}\in\mathbf{V}$ and $\lambda\in[0,1]$. Then the expected revenue of the seller is equal to $\lambda Z^*\in[0,Z^*]$.

Hence, the mediator–optimal mechanism on the set of all feasible mechanisms must satisfy conditions of Lemma 2 and select the seller's expected revenue that solves the following maximization problem,

$$\max_{U_0^*\in[0,Z^*]} \left(Z^* - U_0^*\right)H\left(U_0^*\right). \tag{13}$$

That is, the optimal choice of the seller's expected revenue U_0^* conditional on the event that the auction occurs will balance two opposite forces: a higher seller's (conditional) revenue leads on the one hand to a lower revenue for the mediator if the auction occurs, $Z^* - U_0^*$, but on the other hand to a greater probability, $H(U_0^*)$, that the seller will auction the object.

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5. Implementation

Now we will demonstrate that a mediator–optimal mechanism is implementable by a *closing-fee auction*. In every period, the mediator runs a Vickrey auction with a reserve price. The seller submits a reserve price, r, and every bidder submits a bid. The winning bidder (if any) pays the *closing price* equal to greater of the second highest bid and the reserve price. If the object is sold, the mediator collects a *closing fee*, a fixed percentage $\mu\in[0,1]$ from the closing price. Namely, if there is a winning bidder and the closing price is equal to ρ, then the mediator leaves $\mu\rho$ for herself and passes $(1-\mu)\rho$ to the seller.

Let $\mu^*=1-U_0^*/Z^*$, where Z^* is given by Lemma 2 and U_0 is the solution of the optimization problem (13).

Theorem 1. *The closing-fee auction with closing fee μ is mediator–optimal.*

Proof. See the Appendix.□

The intuition behind this result is as follows.[8] In our model, the seller's optimal strategy stipulates to always re-auction the object if it is not sold. Therefore, the relevant valuation of the object is the expected value derived from future resales, and *not* the use value derived from its consumption. Thus, in contrast to Myerson and Satterthwaite (1983), our mediator–optimal mechanism need not make use of the seller's private information.

Note that a Vickrey auction with reserve price is not a direct mechanism per se. A seller chooses the reserve price that maximizes *his own* expected revenue, which need not be equal to the one that is optimal for the mediator. The problem that we tackle here is a design of a fee scheme that provides the seller an incentive to choose the reserve price which is optimal for the mediator. It turns out that the closing-fee auction does the job for an appropriate choice of a closing fee. In this auction the seller and the mediator receive fixed percentages of expected revenues that do not vary with time, so the seller's incentives are perfectly aligned with the mediator's, and, consequently, the seller chooses the mediator–optimal reserve price.

The assumption of *anonymity* of traders is a cornerstone of our results, since it allows us to search for an optimal mechanism among stationary ones (i.e., those which do not depend on time or identity of the traders). If this assumption is relaxed, more general mechanisms must be considered. We do not know whether the closing-fee auction remains optimal in this setting. In fact, we suspect that the mediator might find a better mechanism among non-stationary ones. The reason is that in a non-stationary mechanism, the seller's strategy of always re-auctioning the object until it is sold need not be optimal anymore, so his use value may be relevant for the mechanism design problem. Therefore, the mediator has a potential to discriminate sellers and, possibly, to raise a higher revenue than from a stationary, non-discriminatory mechanism. The fact that *eBay*, the dominant player on the auction market, uses identity-dependent mechanisms by offering discounts to sellers for re-auctioning objects under specific circumstances[9] makes this question especially appealing for future research.

Another interesting question is whether our closing-fee auction remains optimal (or close to optimal) if there is a small probability, ε, that after the auction has failed, the seller consumes the object without re-auctioning it. In this case the seller's expected continuation payoff, $U_0^*(v_0,\varepsilon)$, depends on the seller's use value v_0: after an appropriate adjustment of (7), $U_0^*(v_0,\varepsilon)$ is defined as follows,

$$U_0^*(v_0,\varepsilon) = \overline{x}_0(v_0) + \overline{p}_0(v_0)\delta\Big((1-\varepsilon)U_0^*(v_0,\varepsilon) + \varepsilon v_0\Big).$$

An *optimal* mechanism may thus be a function of v_0 which discriminates sellers whose use values are in the neighborhood of the seller's continuation payoff of the original problem, U_0^*. However, observe that $U_0^*(v_0,\varepsilon)$ is continuous in ε and approaches U_0^* as $\varepsilon\to 0$, so the closing-fee auction approaches an optimal one.

6. Conclusion

Our paper describes Internet-style auctions and characterizes optimal mechanisms for a mediator. In stark contrast to Myerson and Satterthwaite (1983), our mediator–optimal mechanism does not make use of the seller's private information and for thisreason admits a simple implementation via the closing-fee auction.

Obviously, there is a number of restrictions that make our results appropriate (such as existence of a large population of potential bidders) and some possibly relevant features of internet auctions are ignored in our model, for example, bidders' costs of search through ads on an auction site, and possible fees charged for increasing visibility of ads (printing in bold font, moving up the list, etc.) connected to that problem. Nevertheless, we believe that our results are relevant in many situations, and this paper presents a good starting point for further research.

Appendix

Proof of Lemma 2

The following lemma is due to Myerson (1981).

Lemma 3. *Let* $(\mathbf{p},\mathbf{x})$ *be a feasible mechanism. Then for every* $i=1,\dots,n$

$$\overline{x}_i(v_i) = v_i\overline{p}_i(v_i) - \int_{\underline{v}}^{v_i} \overline{p}_i(z)f(z)dz - U_i(0), \tag{14}$$

and

$$\int_{\mathbf{V}_{-0}} x_i(\mathbf{v})\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0} = \int_{\mathbf{V}_{-0}} C(v_i)p_i(\mathbf{v})\mathbf{f}(\mathbf{v})d\mathbf{v} - U_i(0). \tag{15}$$

We now prove Lemma 2. Suppose that $v_0\le U_0^*$. Since $U_M(v_0)=Z(v_0)-U_0^*$ and U_0^* is fixed, the mediator who wishes to maximize $U_M(v_0)$ also maximizes $Z(v_0)$. By (10) and Lemma 3 we have

$$Z(v_0)=\int_{\mathbf{V}_{-0}}\left(p_0(\mathbf{v})\delta Z(v_0)+\sum_{i=1}^{n} p_i(\mathbf{v})C(v_i)\right)\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0}-\sum_{i=1}^{n} U_i(0).$$

The individual rationality constraint requires $U_i(0)\ge 0$, and in the optimal mechanism it is binding, hence $U_i(0)=0$, $i=1,\dots,n$. We now find $\mathbf{p}$ that yields the maximum value of $Z(v_0)$. Clearly, for every $\mathbf{v}$, the optimal $\mathbf{p}(\mathbf{v})$ must assign probability one to bidder i with the highest virtual value $C(v_i)$ if it exceeds $\delta Z(v_0)$, and otherwise probability one to the seller,

$$\mathbf{p}(\mathbf{v})\in \underset{p'\in\Delta^{n+1}}{arg\,max}\left\{p_0'\delta Z(v_0) + \sum_{i=1}^{n} p_i'C(v_i)\right\}, \quad \mathbf{v}\in\mathbf{V}. \tag{16}$$

Thus we obtain

$$Z(v_0) = \int_{\mathbf{V}_{-0}} max\left\{\delta Z(v_0), \max_{i=1,\dots,n} C(v_i)\right\}\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0}. \tag{17}$$

Consequently, $Z_0(v_0)$ is a solution of the above equation on $[\underline{v},\overline{v}]$ and, moreover, it is independent of v_0. We now show that there exists a unique solution. First, note that the right-hand side of (17) is a continuous function of $Z(v_0)$ and it is always between v and v for any $Z(v_0)\in[\underline{v},\overline{v}]$. Hence, by the Brouwer fixed point theorem, a

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[8] We are grateful to an anonymous referee for helpful insights on this issue.

[9] See http://pages.ebay.com/help/sell/relist.html. We thank an anonymous referee for pointing out this fact.

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solution exists. Next, subtracting $\delta Z(v_0)$ from both sides of (17) and replacing $Z(v_0)$ by z yields

$$(1-\delta)z = \int_{\mathbf{V}_{-0}} max\left\{0, \max_{i=1,\ldots,n} C(v_i)-\delta z\right\}\mathbf{f}_{-0}(\mathbf{v}_{-0})d\mathbf{v}_{-0}.$$

The left-hand side of the above equation is strictly increasing in z and the right-hand side is weakly decreasing, thus there exists only one solution.

We denote the solution of Eq. (17) by Z^*. Substituting $Z(v_0)$ by Z^* in (16) and (17) yields, respectively, parts (ii) and (i) of Lemma 2.

It remains to prove part (iii). By Lemma 3 for every $i = 1,2,\ldots,n$, x_i must satisfy

$$\overline{x}_i(v_i) = v_i\overline{p}_i(v_i)-\int_{\underline{v}}^{v_i} \overline{p}_i(z)f(z)dz-U_i(0)$$

and the constraint $U_i(0)\geq 0$ is binding in the optimal mechanism, hence $U_i(0)=0$. Finally, by (7) the seller's expected revenue from auctioning the object is equal to U_0^* if and only if $\overline{x}_0(v_0) = U_0^*(1-\delta\overline{p}_0(v_0))$.

Proof of Theorem 1

We know from Section 4 that a feasible mechanism $(\mathbf{p},\mathbf{x})$ is optimal if it is the Myerson's (1981) optimal auction with the seller's continuation revenue equal to δZ^* and with x_0 satisfying $\overline{x}_0(v_0) = U_0^*(1-\delta\overline{p}_0(v_0))$, $v_0\in[\underline{v},\overline{v}]$. Here, Z^* is given by Lemma 2, part (i), and it is the highest joint revenue of the seller and the mediator that can be attained among feasible mechanisms, conditional on the object being auctioned; U_0^* is the solution of the maximization problem (13) and it is the value of the seller's expected revenue (conditional on the object being auctioned) which maximizes the mediator's (unconditional) expected revenue.

Let $(\mathbf{p},\mathbf{x})$ be the closing-fee auction with the fee $\mu^* = 1-U_0^*/Z^*$. This is a Vickrey auction with n bidders where the seller obtains a fraction $1-\mu^*$ of the total revenue and chooses a reserve price that maximizes his own revenue. Hence it is the Myerson's (1981) optimal auction whenever the seller's reserve price r^* satisfies (e.g., Krishna 2002)

$$r^*-\frac{1-F\left(r^*\right)}{f\left(r^*\right)} = \delta Z^*. \qquad (18)$$

We will show that, with the given choice of a closing fee, the seller's expected revenue from auction is precisely U_0^* and the reserve price that maximizes the seller's revenue coincides with the reserve price r^* that is optimal for the mediator.

First, since the total revenue from the auction is Z^* and the seller obtains fraction $1-\mu^*$ of the revenue, it follows that the seller's expected revenue from the (repeated) auction, is equal to $(1-\mu^*)Z^* = U_0^*$.

Second, in the closing-fee auction for every reserve price r denote by $\pi(r)$ the *probability that the object is sold* and by $\rho(r)$ the *expected closing price*. In other words, $\rho(r)$ is the expected payment of the winning bidder conditional on the event that the object is sold. Then the optimal expected revenue of the seller is given by the following:

$$U_0^* = \max_{r\geq \underline{v}}\left\{\left(1-\mu^*\right)\pi(r)\rho(r) + (1-\pi(r))\delta U_0^*\right\}.$$

Dividing both sides of the above equation by $(1-\mu^*)$, we obtain the following:

$$\frac{U_0^*}{1-\mu^*} = \max_{r\geq \underline{v}}\left\{\pi(r)\rho(r) + (1-\pi(r))\frac{\delta U_0^*}{1-\mu^*}\right\}.$$

The reserve price $r = r(U_0^*)$ that maximizes the above expression for any given U_0^* (e.g., Krishna, 2002) satisfies as follows:

$$r-\frac{1-F(r)}{f(r)} = \frac{\delta U_0^*}{1-\mu^*}. \qquad (19)$$

As by assumption $r-\frac{1-F(r)}{f(r)}\equiv C(r)$ is strictly increasing, the optimal reserve price is unique. Since $\mu^* = 1-U_0^*/Z^*$, the right-hand side of (19) is equal to $\frac{\delta U_0^*}{U_0^*/Z^*} = \delta Z^*$. Hence, Eqs. (18) and (19) are identical, and so are their solutions, $r(U_0^*) = r^*$.

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